Lesson 20 (3.11)

Today: * Related Rates (part 2)

Office Hours: MWF: 2:45PM – 4:15PM, MATH 842.

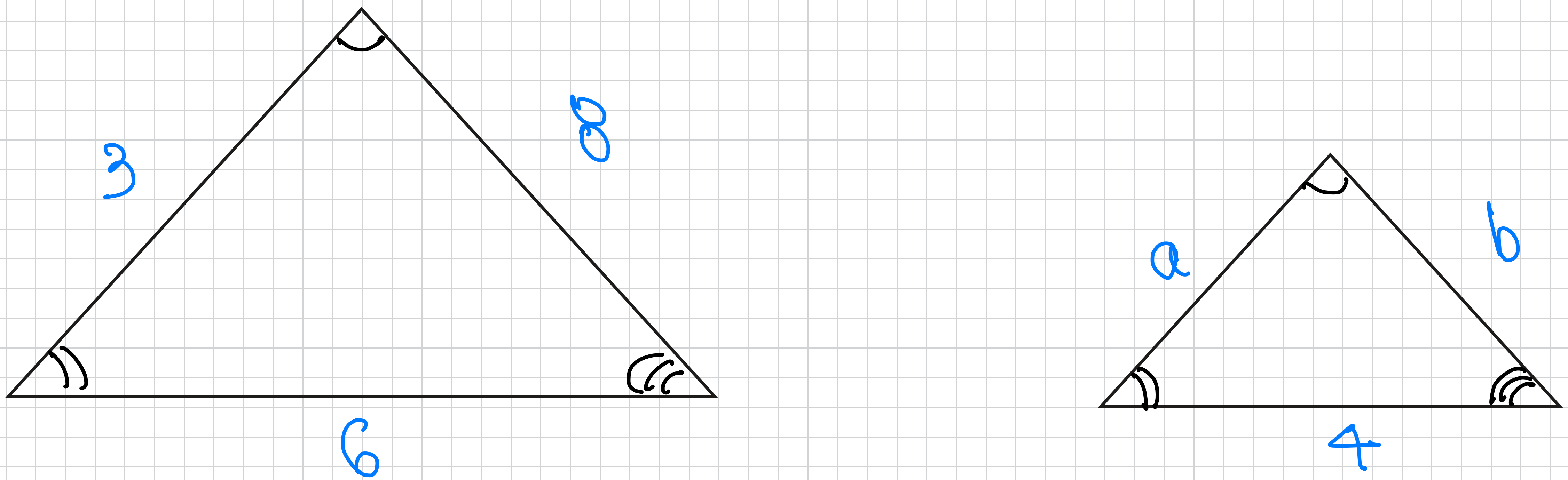
Announcements: * Exam Grades are posted (Average = 70, Median = 76)

* HW Due: Lesson 20,21 – Tuesday

* Quiz 12: Lesson 18 – Tuesday

* Quiz 13: Lesson 19,20,21 Thursday

Warmup:



Find a & b values, if the two triangles are Similar

Lengths of sides are proportional

$$\frac{3}{6} = \frac{a}{4} \Rightarrow a = 2 \quad , \quad \frac{8}{6} = \frac{b}{4} \Rightarrow b = \frac{32}{6}$$

Related Rates

* Word problems
* Variable change with time.

1. Read Carefully

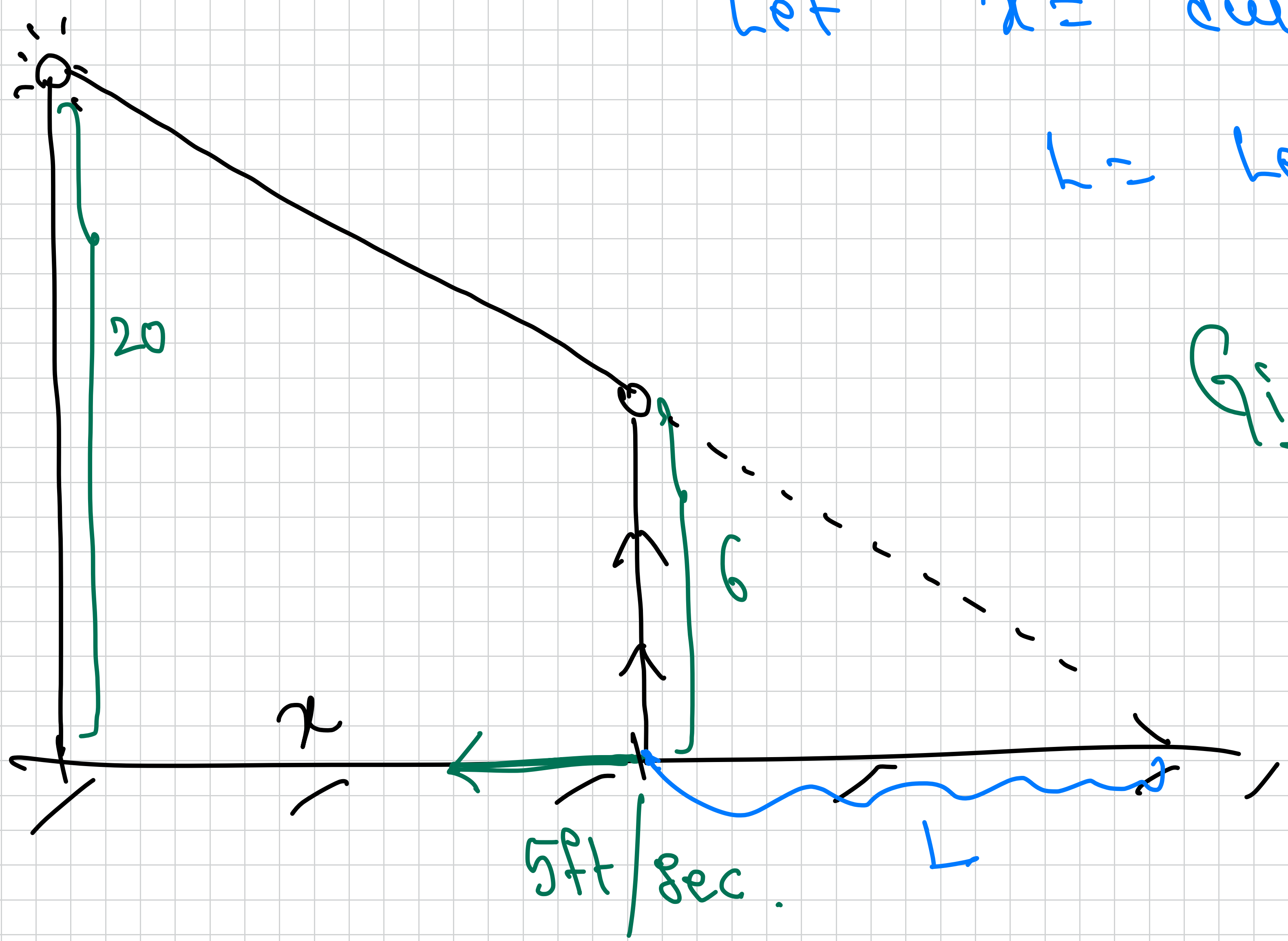
2. Draw a picture & label all things that change with time as variables

3. Write what is given/known
   what to find/unknown

4. Equation relating unknowns & known

5. $\frac{d}{dt}$ of equation, plug in known
   find unknown.

A 6ft tall man walks towards a street light on a post 20ft above the ground at a rate of 5ft/sec. Find the rate of change of the length of his shadow when he is 24ft from the base of the lamp post.



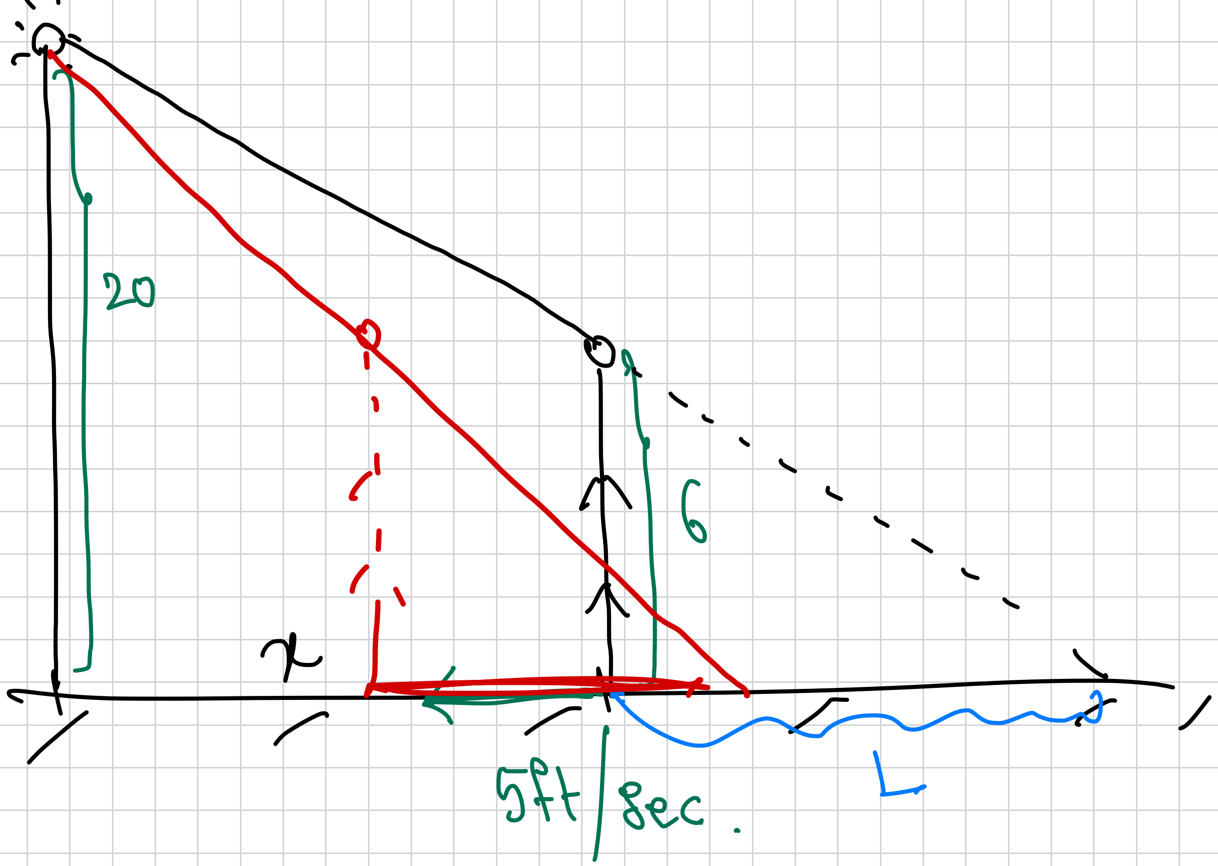
Let x = distance of the person from Lamp post

L = Length of the shadow

Given: $\frac{dx}{dt} = -5$ ft/sec

Find $\frac{dL}{dt}$ when $x = 24$ ft

Equation relating x & L.



$$\frac{x+L}{20} = \frac{L}{6}$$

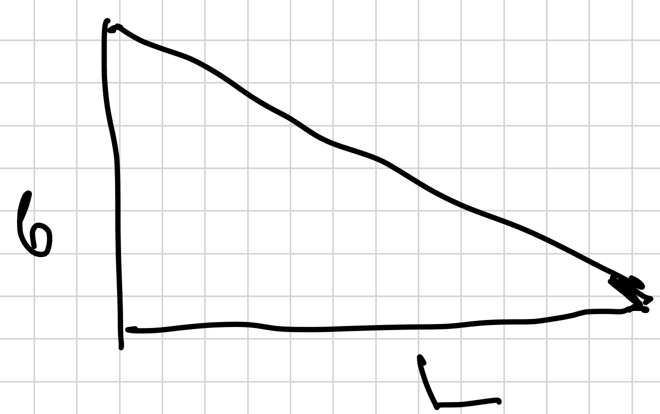
$$6(x+L) = 20L$$

$$\boxed{6x = 14L}$$

$\frac{d}{dt}$ on both sides

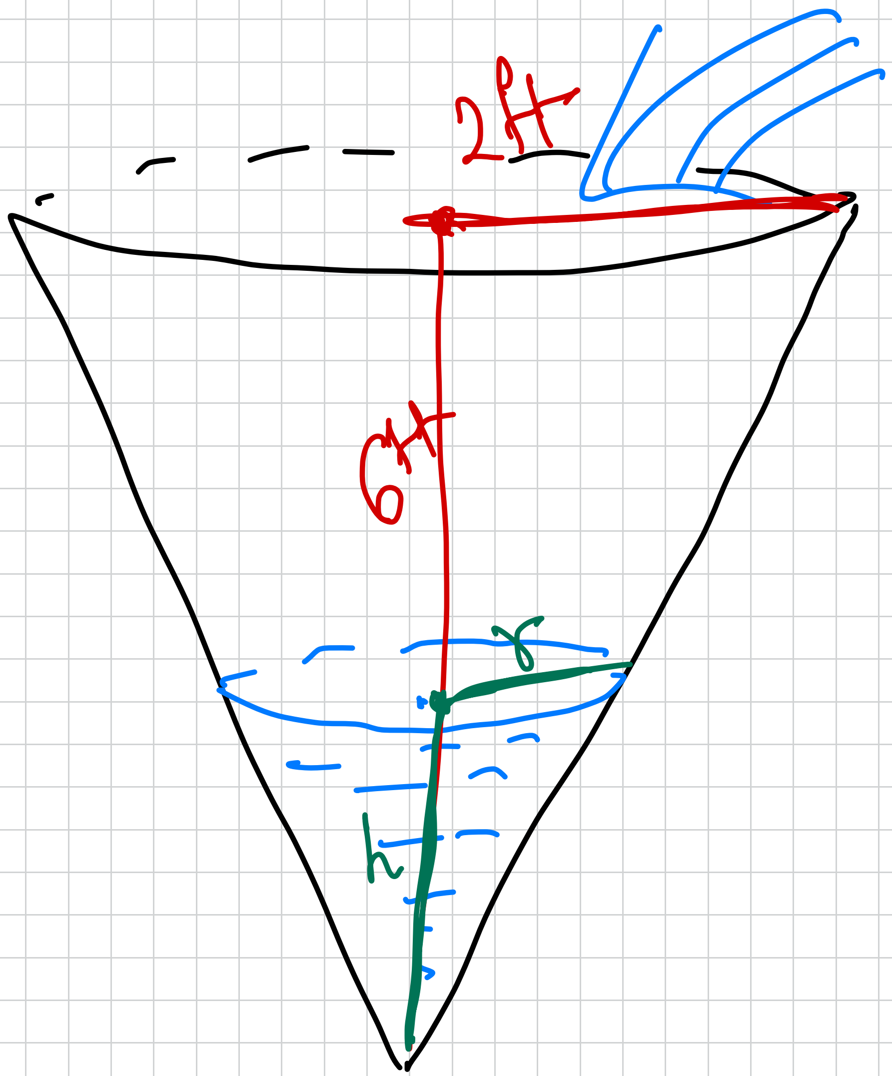
$$6 \cdot \frac{dx}{dt} = 14 \frac{dL}{dt} \; ?$$

$= -5$ ft/sec.



$$\boxed{\frac{dL}{dt} = \frac{-30}{14} \text{ ft/sec}}$$

Water is poured into an inverted conical tank at a rate of $2/3\ ft^3/min$.
If the tank is 6ft tall and has radius 2ft, how fast does the water level rise when the water is 4ft deep?



$2/3\ ft^3/min$

Let h = height of water level from the bottom of cone

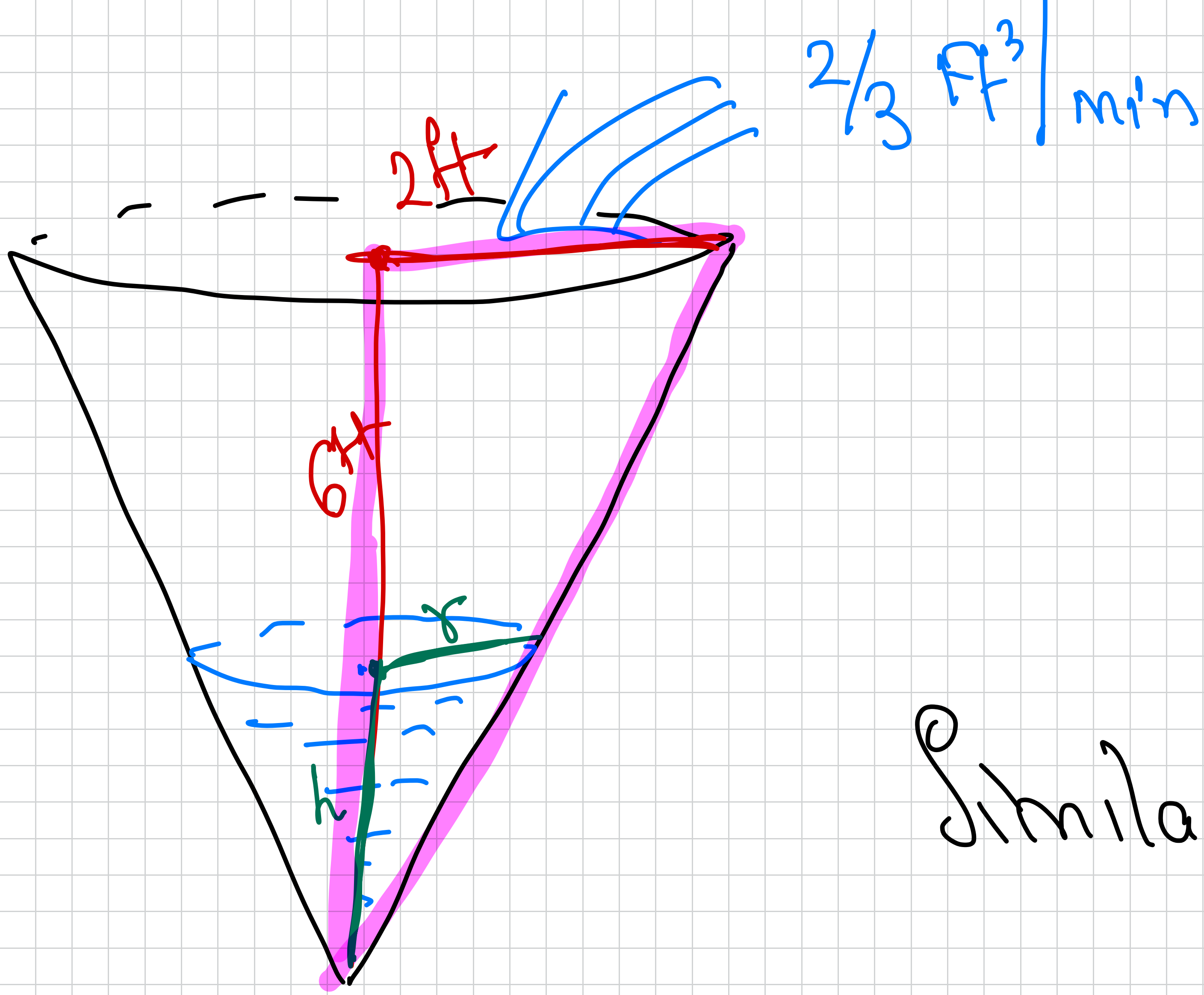
r = radius at the top surface of water

V = volume of water in tank

$$\frac{dV}{dt} = \frac{2}{3}\ ft^3/min$$

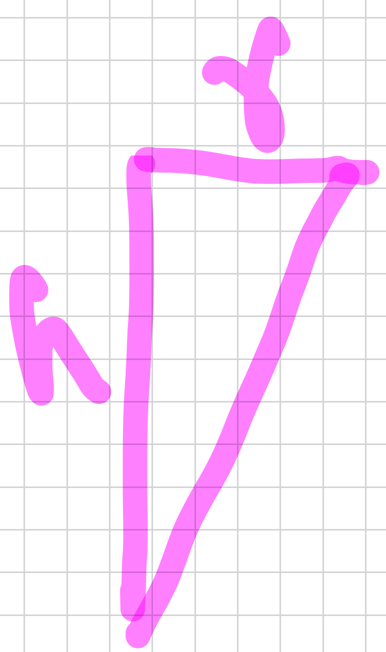
to find $\frac{dh}{dt}$, when $h = 4ft$.

Equation: $V = \frac{1}{3}\pi r^2 h$



Equation: $V = \frac{1}{3}\pi r^2 h$

get rid of r some how!!



Similar triangles $\frac{2}{6} = \frac{r}{h}$

$r = \frac{h}{3}$

$V = \frac{1}{3}\pi\left(\frac{h}{3}\right)^2 \cdot h$

$$\boxed{V = \frac{\pi}{27}h^3}$$

$V = \frac{\pi}{27} h^3$

$\frac{d}{dt}$ on both sides, find $\frac{dh}{dt}$ when $h = 4$

given $\frac{dV}{dt} = \frac{2}{3}$

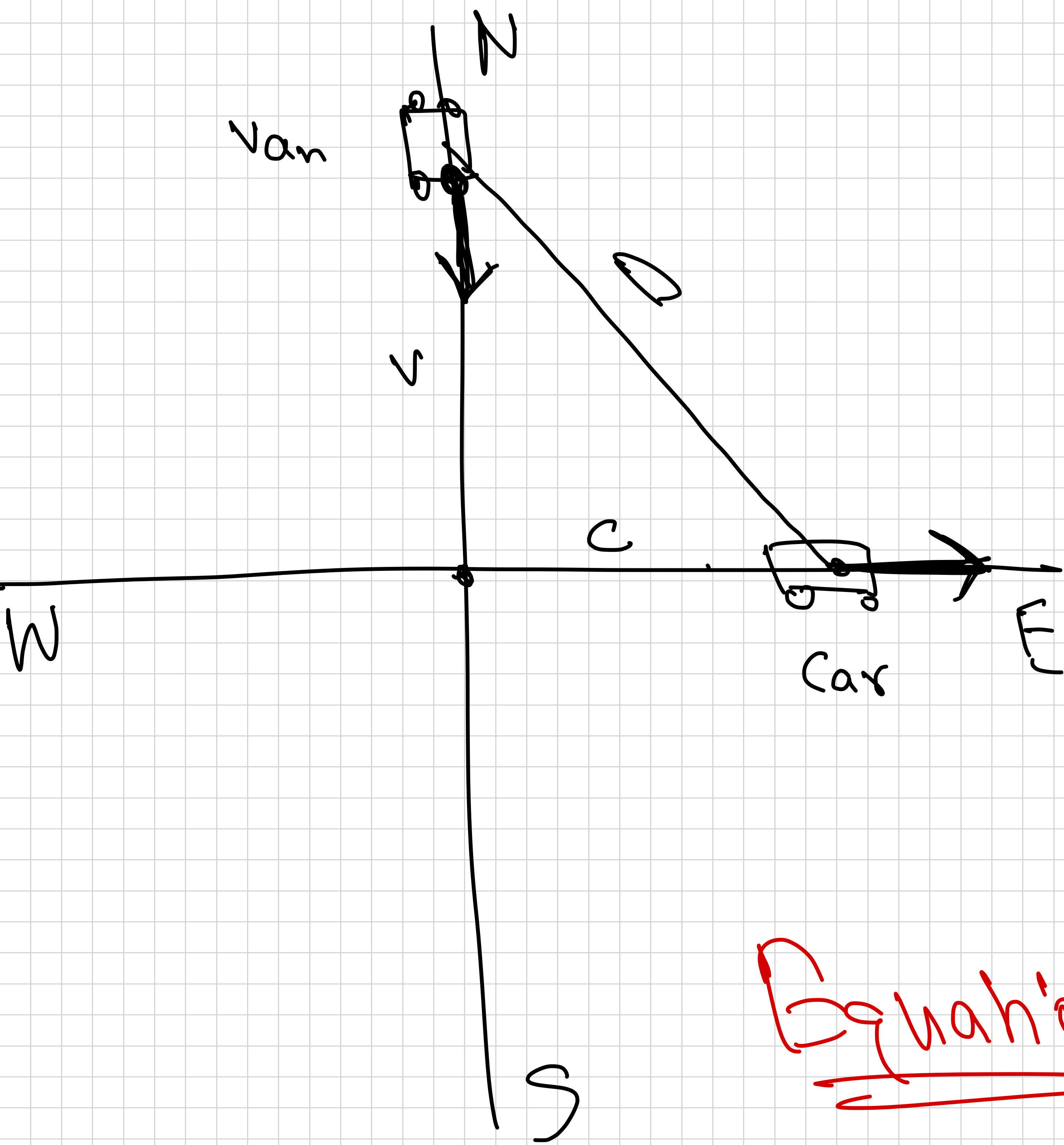
$\frac{2}{3}$

$$\frac{dV}{dt} = \frac{\pi}{27} \frac{d}{dt} h^3 = \frac{\pi}{27} * 3\underset{h=4}{h^2} \cdot \frac{dh}{dt} \; ?$$

$$\frac{2}{3} ft^3/min = \frac{\pi}{\cancel{27}\,9} * \cancel{3} * 16 \; ft^2 \cdot \frac{dh}{dt} = \frac{16\pi}{9} ft^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{\cancel{2}\; ft^{\cancel{3}}}{3 \; min} * \frac{\cancel{9}^{3}}{\cancel{16}_{8}\,\pi\, \cancel{ft^2}} = \frac{3}{8\pi} ft/min.$$

A van travels towards an intersection from the north while a car travels away from the intersection going east. When the van is 0.6 mi north of the intersection and the car is 0.8 mi east of the intersection, the distance between them is increasing at 20mph. If at that instant the van is moving at 60mph, what is the speed of the car?



v = distance of van from intersection

c = distance of car from intersection

D = distance b/w van & car

$\frac{dD}{dt} = 20$ mph when $c = 0.8$, $v = 0.6$

$\frac{dv}{dt} = -60$mph, find $\frac{dc}{dt}$

Equation!

$c^2 + v^2 = D^2$

$$C^2 + V^2 = D^2$$

$\frac{d}{dt}$ on both sides

$$2C \cdot \frac{dc}{dt} + 2V \frac{dv}{dt} = 2D \frac{dD}{dt}$$

$c = 0.8$?

0.6 -60 mph

$= 20$ mph

Not given but we can find it using $c^2 + V^2 = D^2$

$(0.8)^2 + (0.6)^2 = D^2$

$\boxed{D = 1}$

$$(\cancel{2} * 0.8)\frac{dc}{dt} - (\cancel{2} * 0.6 * 60) = (\cancel{2} * 1 * 20)$$

$$0.8 \frac{dc}{dt} = 20 + 36 = 56$$

$$\frac{dc}{dt} = \frac{56}{0.8} = 70 \text{ mph}$$